Chapter 7 Straight Line Graph

1. **Solutions by accurate drawing will not be accepted.**

The points A and B have coordinates (- 2, 4) and (6, 10) respectively.

(a) Find the equation of the perpendicular bisector of the line AB, giving your answer in the form $ax+by+c = 0$, where a, b and c are integers.

$$m_{AB} = \frac{4-10}{-2-6} = \frac{-6}{-8} = \frac{3}{4}$$

[4]

$$m_{\perp} = -\frac{4}{3}$$

$$\text{midpoint} = \left(\frac{-2+6}{2}, \frac{4+10}{2}\right)$$

$$= (2, 7)$$

$$y = -\frac{4}{3}x + c$$

$$7 = -\frac{8}{3} + c$$

$$c = 7 + \frac{8}{3} = \frac{29}{3}$$

$$3y = -4x + 29$$

$$3y + 4x - 29 = 0$$

The point C has coordinates (5, p) and lies on the perpendicular bisector of AB.
(b) Find the value of p.

$$3p + 20 - 29 = 0$$

$$3p = 9$$

$$p = 3$$

[1]

It is given that the line AB bisects the line CD.
(c) Find the coordinates of D.

midpt of AB = midpt of CD

$$(2, 7) = \left(\frac{x+5}{2}, \frac{y+3}{2}\right)$$

[2]

$$\frac{x+5}{2} = 2 \qquad \frac{y+3}{2} = 7$$

$$x + 5 = 4 \qquad y + 3 = 14$$

$$x = -1 \qquad y = 11$$

$$\therefore D(-1, 11)$$

2. **Solutions to this question by accurate drawing will not be accepted.**

The points *A* and *B* are (4, 3) and (12, −7) respectively.

a. Find the equation of the line *L*, the perpendicular bisector of the line *AB*.

[4]

$m = \frac{-7-3}{12-4} = \frac{-10}{8} = -\frac{5}{4}$

$m_2 = \frac{4}{5}$

$\text{midpt} = \left(\frac{4+12}{2}, \frac{3-7}{2}\right) = (8, -2)$

$y = \frac{4}{5}x + c$

$-2 = \frac{32}{5} + c$

$c = \frac{-10-32}{5} = -\frac{42}{5}$ $\quad \therefore 5y = 4x - 42$

b. The line parallel to *AB* which passes through the point (5, 12) intersects *L* at the point *C*. Find the coordinates of *C*.

[4]

$m = -\frac{5}{4}$ $\quad y = -\frac{5}{4}x + c$

$12 = -\frac{25}{4} + c$

$c = \frac{48+25}{4}$

$= \frac{73}{4}$

$4y = -5x + 73 \quad \times 4$

$5y = 4x - 42 \quad \times 5$

$16y = -\cancel{20x} + 292$

$25y = \cancel{20x} - 210$

$41y = 82$

$y = 2$

$\therefore C(13, 2)$

$4x - 42 = 5y$

$4x = 10 + 42$

$4x = 52$

$x = 13$

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3. **Solutions to this question by accurate drawing will not be accepted.**

Find the equation of the perpendicular bisector of the line joining the points (4, -7) and (-8, 9).

[4]

$$m = \frac{9+7}{-8-4} = \frac{16}{-12} = -\frac{4}{3}$$

$$m_{\perp} = \frac{3}{4}$$

$$\text{midpt } \left(\frac{4-8}{2}, \frac{-7+9}{2}\right) = (-2, 1)$$

$$y = \frac{3}{4}x + c$$

$$1 = -\frac{3}{2} + c$$

$$c = \frac{2}{2} + \frac{3}{2} = \frac{5}{2}$$

$$y = \frac{3}{4}x + \frac{5}{2}$$

$$4y = 3x + 10$$

4. (a) Find the equation of the perpendicular bisector of the line joining the points (12, 1) and (4, 3), giving your answer in the form $y = mx + c$.

[5]

$m = \frac{3-1}{4-12} = \frac{2}{-8} = -\frac{1}{4}$

$m_\perp = 4$

midpt = $(8, 2)$

$y = 4x + c$

$2 = 32 + c$

$c = -30$

$y = 4x - 30$

(b) The perpendicular bisector cuts the axes at points A and B. Find the length of AB.

[3]

$y = 4x - 30$

$x = 0,\ y = -30 \quad (0, -30)$

$y = 0,\ x = \frac{15}{2} \quad (\frac{15}{2}, 0)$

length $AB = \sqrt{(\frac{15}{2})^2 + (-30)^2}$

$= \frac{15\sqrt{17}}{2}$

$= 30.9$

5. The line $y = 5x + 6$ meets the curve $xy = 8$ at the points A and B.

(a) Find the coordinates of A and of B.

$xy = 8 \longrightarrow y = \frac{8}{x}$ [3]

$x(5x+6) = 8$

$5x^2 + 6x - 8 = 0$

$(5x-4)(x+2) = 0$

$x = \frac{4}{5} \qquad x = -2$

$y = 10 \qquad y = -4$

$A\left(\frac{4}{5}, 10\right)$

$B(-2, -4)$

(b) Find the coordinates of the point where the perpendicular bisector of the line AB meets the line $y = x$.

$m = \dfrac{-4-10}{-2-\frac{4}{5}} = \dfrac{-14}{\frac{-14}{5}} = 5 \qquad \text{midpt} = \left(-\frac{3}{5}, 3\right)$ [5]

$m_{\perp} = -\frac{1}{5}$

$y = -\frac{1}{5}x + c$

$3 = \frac{3}{25} + c$

$c = \dfrac{75-3}{25} = \dfrac{72}{25}$

$y = -\frac{1}{5}x + \frac{72}{25}$

$25x = -5x + 72$

$30x = 72$

$x = \dfrac{\cancel{72}^{24}}{\cancel{30}_{10}} = 2.4$

$y = 2.4$

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6. Variables *x* and *y* are such that, when $lg\ y$ is plotted against x^3, a straight line graph passing through the points (6, 7) and (10, 9) is obtained. Find *y* as a function of *x*.

[4]

$$m = \frac{9-7}{10-6} = \frac{2}{4} = \frac{1}{2}$$

$$\lg y = \frac{1}{2}x^3 + c$$

$$7 = 3 + c$$

$$c = 4$$

$$\lg y = \frac{1}{2}x^3 + 4$$

$$y = 10^{\frac{1}{2}x^3 + 4}$$

7. Variables *x* and *y* are such that, when $\sqrt[4]{y}$ is plotted against $\frac{1}{x}$, a straight line graph passing through the points (0.5, 9) and (3, 34) is obtained. Find *y* as a function of *x*.

[4]

$$m = \frac{34-9}{3-0.5} = 10$$

$$\sqrt[4]{y} = 10 \times \frac{1}{x} + c$$

$$9 = 10 \times \frac{1}{2} + c$$

$$9 = 5 + c$$

$$c = 4$$

$$\sqrt[4]{y} = 10 \times \frac{1}{x} + 4$$

$$y = \left(10 \times \frac{1}{x} + 4\right)^4$$

8. Variables *x* and *y* are connected by the relationship $y = Ax^n$, where *A* and *n* are constants.

(a) Transform the relationship $y = Ax^n$ to straight line form.

$\ln y = \ln Ax^n$

$= \ln A + n \ln x$

[2]

When $ln\,y$ is plotted against $ln\,x$ a straight line graph passing through the points (0, 0.5) and (3.2, 1.7) is obtained.

(b) Find the value of *n* and of *A*.

$m = \frac{1.7 - 0.5}{3.2 - 0} = \frac{1.2}{3.2} = \frac{12}{32} = \frac{3}{8}$

[4]

$\ln y = m \ln x + c$

$0.5 = \frac{3}{8}(0) + c$

$c = 0.5$

$\ln y = \frac{3}{8} \ln x + \frac{1}{2}$

$n = \frac{3}{8}$

$\ln A = \frac{1}{2}$

$A = e^{1/2}$

(c) Find the value of *y* when *x* = 11.

$y = Ax^n$

$= e^{1/2} \times 11^{3/8}$

$= 4.05$

[2]

9. It is known that $y = A \times 10^{bx^2}$, where *A* and *b* are constants. When $lg\ y$ is plotted against x^2, a straight line passing through the points (3.63, 5.25) and (4.83, 6.88) is obtained.

a. Find the value of *A* and of *b*. [4]

$$\lg y = \lg A \times 10^{bx^2} \qquad m = \frac{6.88 - 5.25}{4.83 - 3.63} = \frac{163}{120}$$

$$\lg y = \lg A + bx^2$$

$$\lg y = \frac{163}{120} x^2 + C$$

$$5.25 = \frac{163}{120} \times 3.63 + C$$

$$5.25 = 4.93075 + C$$

$$C = 0.319$$

$$\lg y = 1.36x^2 + 0.319$$

$$b = 1.36$$

$$\lg A = 0.319$$

$$A = 2.08$$

Using your values of *A* and *b*, find

b. the value of *y* when *x* = 2, [2]

$$\lg y = 1.36x^2 + 0.319$$

$$= 1.36 \times 4 + 0.319$$

$$= 5.759$$

$$y = 574116$$

c. the positive value of *x* when *y* = 4. [2]

$$\lg 4 = 1.36x^2 + 0.319$$

$$1.36x^2 = 0.2831$$

$$x^2 = 0.2081$$

$$x = 0.456$$